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On the real crystal rhombododecahedra

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In this paper, all the real crystal rhombododecahedra (625 in total) are enumerated and characterized by facet symbols and symmetry point groups. All the dodecahedra (34 in total) are drawn. Nine combinatorially different types are found among the garnets from Mt Makzapakhk, Kola Peninsula.

1. Introduction

This paper is a continuation of our investigation of the real crystal forms that appear to be interesting from both mathematical and crystallographic points of view (Voytekhovsky, 2002). To generalize, we define a real crystal form as any polyhedron bounded, at least, by some of the planes of a given ideal crystal form in a standard orientation with arbitrary distances from the origin of the coordinates. A generating algorithm is explained by Voytekhovsky (2002). Among the 47 ideal crystal simple forms, 11 do not lead to real varieties. Of the 36 remaining ideal crystal simple forms, 17 were investigated by Voytekhovsky & Stepenshchikov (2004). In the present paper, all the real crystal rhombododecahedra (625 in total) are enumerated and characterized for the first time by their facet symbols and symmetry point groups. The theory is applied to the interpretation of the morphology of garnet crystals from Mt Makzapakhk, Kola Peninsula.

2. Real crystal rhombododecahedra

The real crystal rhombododecahedra are classified in Table 1 by their automorphism group orders (a.g.o.), *i.e.* isomorphism group orders of abstract point groups, symmetry point groups (s.p.g.), number of facets and facet symbols. The latter give the numbers of 3-, 4-, ..., *n*-gonal facets of the polyhedron in sequence.

The number of real crystal rhombododecahedra decreases with increasing automorphism group order. The dependence on the number of facets is more complicated; the number of real crystal rhombododecahedra is maximum for 10-hedra and slightly decreases for 12-hedra. The latter result from the ideal rhombododecahedron if its facets are displaced a little from the ideal positions (Fig. 1). Just these types of crystals appear to form in an environment that has a low diffusion gradient. Some of these crystals have already been found in nature. Goldschmidt (1918) reported the following morphological types of garnets: an ideal form [0,12] $m\bar{3}m$ (Taf. 55, Fig. 1), a combinatorially ideal form elongated along the 3 symmetry axis (Taf. 61, Fig. 92) and a form [0804] 4/*mmm* with a belt of hexagons around the 4 symmetry axis (Taf. 57, Fig. 42). Similar forms are also reported for diamonds (Goldschmidt, 1916): an ideal form (Taf. 27, Fig. 13; Taf. 28, Fig. 23; Taf. 29, Fig. 61; Taf. 34, Fig. 150), a form elongated along the 3 symmetry axis (Taf. 28, Figs. 26, 27), a form with a belt of hexagons around the 4 symmetry axis (Taf. 40, Fig. 264) and a rare form (Taf. 40, Fig. 261) belonging to [084] *mm*2 or [084] $\bar{4}2m$ types depending on the face relation at the invisible side of the

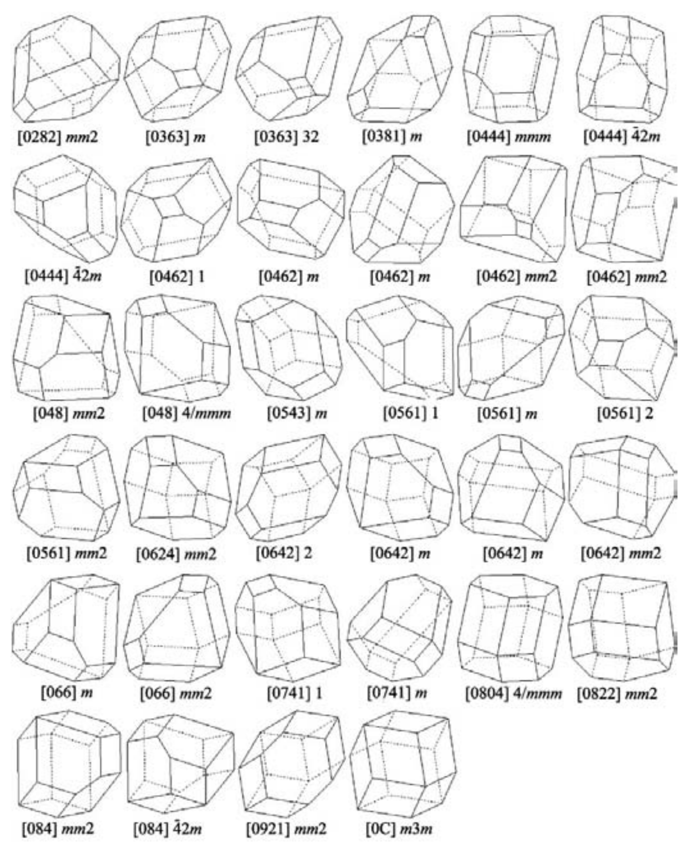

Figure 1
The fully faceted real crystal rhombododecahedra.

Table 1
Statistics of the real rhombododecahedra.

a.g.o. →		1	2			4			6	8		12	16	48
s.p.g. →		1	m	2	$\bar{1}$	$mm2$	$2/m$	222	32	mmm	$\bar{4}2m$	$\bar{3}m$	$4/mmm$	$m\bar{3}m$
4-hedron	[4]										1			
5-hedra,	[23]		1	3										
5	[41]					1								
6-hedra,	[06]	3		2		2	1	1				1		
19	[222]	3		1		1								
	[24]	1		1		1								
	[321]		1											
7-hedra,	[052]	9	2	4										
56	[133]	7	3											
	[151]	12	2											
	[2221]			2										
	[2302]			1										
	[232]	1		1										
	[2401]					1								
	[25]	1	3	1										
	[3031]		1											
	[313]	1	1											
	[331]	1	1											
	[43]		1											
8-hedra,	[044]	17		4										
120	[0602]	1		1		1	2	1						
	[062]	9	4	2										
	[08]		1											
	[1331]	8	2											
	[1412]	1												
	[143]	10												
	[1511]	6	2											
	[161]	4												
	[206]						1							
	[2141]			1		1								
	[2222]	1		1		1								
	[224]	3	1											
	[2321]	6	1											
	[242]	4		1	1		1							
	[2501]	2	1	1		1								
	[323]	1	2											
	[341]	1												
	[4004]										1			
	[4022]			1		1								
	[4121]	1	1											
	[4301]		1											
	[44]						1			1	1			
	[503]		1											
	[8]												1	
9-hedra,	[036]	6	3	2					1					
153	[0441]	9	1	2										
	[0522]	6		1										
	[054]	12	5	1										
	[0603]								1					
	[0621]	14		1										
	[072]	4	1	3										
	[0801]	2	1											
	[09]								1					
	[1251]	2												
	[1332]	6												
	[135]	7	1											
	[1431]	14	1											
	[1512]	1	1											
	[153]	12	2											
	[1611]	5												
	[171]	3	1											
	[2304]			1										

Table 1 (continued)

a.g.o. →		1	2			4			6	8		12	16	48
s.p.g. →		1	m	2	$\bar{1}$	$mm2$	$2/m$	222	32	mmm	$\bar{4}2m$	$\bar{3}m$	$4/mmm$	$m\bar{3}m$
	[2322]	2		1										
	[234]	3												
	[2421]	4												
	[252]	1	3											
	[2601]	1												
	[27]		1	1										
	[333]	1												
	[45]					1								
10-hedra,	[028]	1	1	2										
162	[0361]	4	2											
	[0442]	7		6	1									
	[046]	9	2	1										
	[0523]		1	2										
	[0541]	19	3											
	[0604]							1						
	[0622]	4	1	3										
	[064]	12	1	4		1	2							
	[0721]	8		2										
	[0802]					1								
	[082]	5		1		1								
	[0901]		1											
	[0,10]			1						1				
	[1252]		2											
	[127]		2											
	[1333]		1											
	[1351]	3	1											
	[1414]		1											
	[1432]	1	4											
	[145]	3	4											
	[1513]		1											
	[1531]	5	2											
	[1612]		1											
	[163]	4	3											
	[1711]	1	1											
	[181]	1	1											
	[226]					2								
	[244]		2			1								
	[262]		1			2								
	[28]					1								
11-hedra,	[0281]	2		1										
75	[0362]	1		1										
	[038]	2	1											
	[0443]	3		1										
	[0461]	9	1	1										
	[0524]			1										
	[0542]	4	2	1										
	[056]	6	3	2										
	[0623]	2												
	[0641]	9		1										
	[0722]	1	1	1										
	[074]	6	2	1		1								
	[0821]	2				1								
	[092]	1	2	1										
	[0,11]					1								
12-hedra,	[0282]					1								
34	[0363]		1						1					
	[0381]		1											
	[0444]									1	2			
	[0462]	1	2			2								
	[048]					1							1	
	[0543]		1											
	[0561]	1	1	1		1								
	[0624]					1								
	[0642]		2	1		1								
	[066]		1			1								
	[0741]	1	1											

Table 1 (continued)

a.g.o. →	1	2			4			6	8		12	16	48
s.p.g. →	1	*m*	2	$\bar{1}$	*mm*2	2/*m*	222	32	*mmm*	$\bar{4}2m$	$\bar{3}m$	4/*mmm*	$m\bar{3}m$
[0804]												1	
[0822]					1								
[084]					1					1			
[0921]					1								
[0,12]													1
	367	115	77	2	35	8	3	4	3	6	1	3	1
Total (625)	367	194			46			4	9		1	3	1

figure. The same forms are reported for crystals of magnetite, pyrite and some other cubic minerals.

3. Garnets from Mt Makzapakhk

Here, we briefly outline the actual occurrence of garnets at Mt Makzapakhk, West Keyvy ridge, Kola Peninsula, in view of a later, more profound, study of the relation of their morphology and the petrology of metamorphic rocks. The deposit was found in the early 1930s. Biotite and garnet-biotite gneisses and schists are widespread there. They are intensively folded and intruded by the alkaline granites. The deposit is located in the lower series of the schists, exposed at the surface as narrow strips of anticline cores, and consists of zones highly (up to 60 vol.%) enriched by garnets. The zones are some meters in width and some dozens of meters in length. The garnets relate to almandine and have sizes of ∼2–4 cm on average; the largest crystals are ∼30 cm (Fig. 2).

About 100 well faceted garnet crystals from Mt Makzapakhk were studied. Most of them are of [0,12] $m\bar{3}m$, [0804] 4/*mmm*, [048] 4/*mmm*, [0921] *mm*2 and [084] *mm*2 types. Only a few crystals are of [0462] *mm*2, [0624] *mm*2, [0462] *m* and [0642] *m* types. Obviously, all of these crystals are fully faceted rhombododecahedra and the most symmetrical forms prevail among them. Hence they appear to arise in an almost isotropic environment in accordance with the dissymmetry principle of Curie (1894).

4. Conclusions

Two ideas form the main motivations for this paper. First, all the real crystal rhombododecahedra are enumerated and characterized for the first time. Second, the concept of real crystal simple forms is shown to be helpful for the description of natural crystals, in our case garnets from Mt Makzapakhk. From the viewpoint of the dissymmetry principle of Curie (1894), these crystals were formed in an almost isotropic environment.

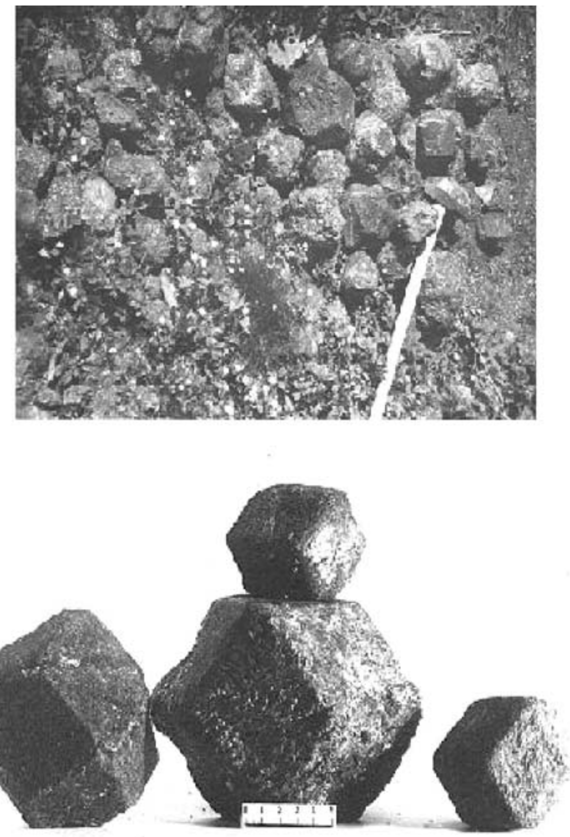

Figure 2
Large garnet crystals from Mt Makzapakhk (Belkov, 1957; Suslova, 1960).

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